

$$\text{var}(I_{y_{DS,t}}) = [a_1, a_2, \dots, a_m] \text{Cov}(GEBV_{y_{DS,t}}) \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} =$$

$$[a_1, a_2, \dots, a_m] \begin{bmatrix} r_{GEBV1,y_{DS,t}}^2 \sigma_{G_{1,D,t}}^2 & r_{GEBV1,y_{DS,t}}^2 r_{GEBV2,y_{DS,t}}^2 \sigma_{G_{2,D,t}} & \dots & r_{GEBV1,y_{DS,t}}^2 r_{GEBV m,y_{DS,t}}^2 \sigma_{G_{m,D,t}} \\ \cdot & r_{GEBV2,y_{DS,t}}^2 \sigma_{G_{2,D,t}}^2 & \dots & r_{GEBV2,y_{DS,t}}^2 r_{GEBV m,y_{DS,t}}^2 \sigma_{G_{m,D,t}} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ symmetry & \cdot & \dots & r_{GEBV m,y_{DS,t}}^2 \sigma_{G_{m,D,t}}^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \dots\dots(a)$$

where $r_{GEBV i,y_{DS,t}}^2$ is the reliability of GEBV in $I_{y_{DS,t}}$ for the i^{th} trait in the dam population of generation t, $\sigma_{G_{i,D,t}}^2$ is the genetic variance

for the i^{th} trait in the dam population of generation t, and $\sigma_{G_{i,j,D,t}}$ is the genetic covariance for traits i and j in the dam population of generation t. In addition,

$$\text{var}(I_{prs_{DS,t}}) = [a_1, a_2, \dots, a_m] \text{Cov}(GEBV_{I_{prs_{DS,t}}}) \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix},$$

where $\text{cov}(GEBV_{I_{prs_{DS,t}}})_t$ is obtained by replacing I_y with I_{prs} in equation (a) and

$$\text{var}(I_{y_{DD,t}}) = [a_1, a_2, \dots, a_m] \begin{bmatrix} r_{GEBV1,y_{DD,t}}^2 \sigma_{G_{1,D,t}}^2 & r_{GEBV1,y_{DD,t}}^2 r_{GEBV2,y_{DD,t}}^2 \sigma_{G_{2,D,t}} & \dots & r_{GEBV1,y_{DD,t}}^2 r_{GEBV m,y_{DD,t}}^2 \sigma_{G_{m,D,t}} \\ \cdot & r_{GEBV2,y_{DD,t}}^2 \sigma_{G_{2,D,t}}^2 & \dots & r_{GEBV2,y_{DD,t}}^2 r_{GEBV m,y_{DD,t}}^2 \sigma_{G_{m,D,t}} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ symmetry & \cdot & \dots & r_{GEBV m,y_{DD,t}}^2 \sigma_{G_{m,D,t}}^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

where $r_{GEBV i,y_{DD,t}}^2$ is the reliability of GEBV in $I_{y_{DD,t}}$ for the i^{th} trait in the dam population of generation t.